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Thermoviscoelastic Characterization of a Composite Solid Propellant Using Tubular Test

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The accurate thermoviscoelastic characterization of a solid propellant is critical for optimum design of grains for high-performance solid motors for space applications. A more realistic experimental method is reported for the characterization of propellants at multiaxial stress conditions using a grain-pressurization test called the tubular test. An idealized cylindrical grain with hydroxyl-terminated-polybutadiene-based propellant is used. From the measured outer radial expansion of the tubular specimen, creep compliance and the relaxation modulus of the propellant are computed based on linear viscoelastic theory. The relaxation-modulus data obtained from the tubular test are shown to be higher than the traditional uniaxial and strip biaxial tests at the same strain level. Different isothermal tubular tests are carried out at various strain levels, and master curves are generated. The tubular test with internal pressurization is found to be an easier, more inexpensive, and realistic method for predicting the relaxation-modulus values of solid propellants and to have great potential for the thermoviscoelastic characterization of solid propellants.

Nomenclature

A	=	constant
a	=	instantaneous inner radius of the tubular specimen, mm
a_T	=	temperature shift factor
b	=	instantaneous outer radius of the tubular specimen, mm
$D(t)$	=	tensile creep compliance, cm^2/kgf
$E(t)$	=	relaxation modulus, kgf/cm^2
$E'(t)$	=	time-dependent modulus, kgf/cm^2
$J(t)$	=	shear creep compliance, cm^2/kgf
k_1, k_2	=	material constants
n	=	time exponent
P_i	=	internal pressure, kgf/cm^2
r	=	radial coordinate
s	=	Laplace transform
T	=	temperature, °C
T_0	=	reference temperature, °C
t	=	time, s
$\Gamma()$	=	gamma function
Δa	=	inner radial expansion of the tubular specimen, mm
Δb	=	outer radial expansion of the tubular specimen, mm
ε_θ^e	=	elastic hoop strain
ε_θ^v	=	viscoelastic hoop strain
$\dot{\varepsilon}$	=	strain rate
ζ	=	reduced time, s
λ	=	initial diameter ratio (b/a) of the tubular specimen
ν	=	Poisson's ratio
σ_r	=	radial stress, kgf/cm^2
σ_z	=	axial stress, kgf/cm^2
σ_θ	=	hoop stress, kgf/cm^2
^	=	Laplace transformation

Introduction

LAUNCH vehicle propellant grain, in general, is a thick-walled hollow cylinder made of viscoelastic materials bonded on its outer periphery to a thin cylindrical case. The configuration of the inner surface of the grain is determined mainly by ballistic considerations.

Solid propellant is considered a viscoelastic material. Viscoelasticity is concerned with materials that exhibit strain rate effects in response to applied stresses. These effects are manifested by the phenomena of creep under constant stress and stress relaxation under constant strain. These time-dependent phenomena may have a considerable effect on the stress distribution developed in a solid-propellant grain subjected to prescribed loads acting successively or simultaneously. The viscoelastic material possesses a characteristic that can be referred to as a "memory effect," that is, the material response is not only determined by the current state of stress, but is also determined by all of the past stress history.

Several loadings may act successively or simultaneously on a rocket propellant grain. Among these, the most important are 1) internal pressure, 2) shrinkage-type thermal loading, 3) thermal gradient loading, 4) thermal transient loading, 5) inertia loads (in flight), 6) local difference of pressure inside the grain, 7) vibrations, 8) residual stresses, and 9) body forces (in storage). In this study, the focus is on internal pressure and the thermal loading. The tubular grains are considered subjected to time-dependent internal pressure loading caused by ignition and subsequent burning. In a certain class of solid motors, unexpected pressure-overshoots/ignition peaks (in excess of the equilibrium chamber pressure) are observed during the starting transient.[1–4] Accurate prediction and control of the ignition peak are still elusive. Therefore, analyzing/reexamining the viscoelastic response of the solid-propellant grain during the internal pressure loading/transient loading at elevated temperature is desirable, and identifying its margin of safety is inevitable to meet mission requirements.[5]

The stress and/or strain at a specific point in the viscoelastic material may vary significantly with time and temperature even though the applied forces are constant. These time–temperature-dependent phenomena may have a considerable effect on the stress distribution developed in a solid-propellant grain subjected to prescribed loads acting successively or simultaneously. To predict the change in stress and strain distribution with time and temperature, thermoviscoelastic characterization is needed.

Most viscoelastic materials exhibit linear or nearly linear behavior over certain ranges of the variables, stress, strain, time, and temperature, although the same material may have a nonlinear behavior over larger ranges of some of the variables. Any demarcation of the boundary between nearly linear (where an assumption of linear behavior is acceptable) and nonlinear is arbitrary. The maximum permissible deviation from linear behavior of a material, which allows

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a linear theory to be employed with acceptable accuracy, depends on the stress distribution, the type of application, and experience. The linear theory of viscoelasticity yields a mathematically tractable representation for stress–strain–time relations, which permits reasonably simple solution for many stress analysis problems. There has been considerable activity in this area over the years to develop new mathematical representations of viscoelastic behavior and new methods for viscoelastic analysis.[5–7]

Generally, viscoelastic stress analysis problems are more involved than elasticity problems due to inclusion of the time variable in the differential equations in addition to the space variables. However, in many problems, where the types of boundary conditions and temperature remain constant in time, the time variable can be removed by employing the Laplace transform. Thus, the problem is converted to an equivalent elastic problem. When a solution for the desired variable has been found in terms of the Laplace transform variable s, the inverse Laplace transformation yields the desired solution in the time variable t for the time-dependent behavior in the viscoelastic problem. This method is called elastic–viscoelastic analogy or the correspondence principle. To solve linear viscoelastic stress analysis problems, the elastic–viscoelastic correspondence principle is by far the simplest method. However, not all linear viscoelastic problems can be solved by this method.

A thorough understanding of the mechanical behavior of the propellant during the burning time is required for grain structural integrity evaluation. The first step in the assessment of the structural integrity of a rocket motor is the laboratory analysis of the propellant-response characteristics and the failure behavior.[8,9] Numerous studies were carried out in earlier investigations of the thermoviscoelastic analysis of solid propellant grains.[10–15] Many experimental methods are available today for the characterization of solid propellants, namely, uniaxial tensile test, strip biaxial test, constant strain failure test, constant stress endurance test, rectangular bond-in-tension, and strain evaluation cylinder. These methods have been used successfully for variety of practical applications for years. However, a more realistic experimental method is needed to recreate the actual stress condition in a solid rocket motor during its mission. In the actual case, multiaxial stress conditions occur in a solid propellant, and strip biaxial tests have been developed for the characterization and the failure prediction.[16] However, even in this test, a pure biaxial stress field is found only at the center of the specimen due to its geometry, and a finite element method is necessary for accurate evaluation. An attempt was made by Nambudiripad and Neis[17] in 1976 to determine the mechanical response of nonlinear viscoelastic solids based on the Frechet expansion. Various numerical studies were conducted during earlier investigations of launch vehicle grains under internal pressure loading.[18–20] However, the variations of the relaxation modulus of solid propellants under varying stress conditions were not reported in these numerical studies.

Several types of multiaxial tests were devised earlier for high-temperature applications.[21] A detailed review on multiaxial testing methods was carried out by Zamrik,[22] but there has been none in the context of time–temperature dependence on solid propellants under multiaxial stress conditions.

In this paper a realistic experimental method, called the tubular test, is reported for the thermoviscoelastic characterization of solid propellants. This method has successfully applied for the viscoelastic characterization of a composite solid propellant.[23,24] An attempt has been made to compare the relaxation-modulus data for the tubular test, uniaxial test and strip biaxial test at the same strain level. This study also aims to make basic improvements in thermoviscoelatic characterization of solid propellants that can be used as a pointer to predict failures in grains subjected to internal pressurization.

Review of Viscoelastic Models

Different viscoelastic models, such as the Maxwell model, Kelvin model, and Burgers model, are available today, but these models and their possible combinations cannot describe the behavior of many viscoelastic materials over a wide range of variables, especially for both large and small values of time. To deal with this, several types of complex mechanical models have been proposed.[6]

The generalized Kelvin model is more convenient than the generalized Maxwell model for viscoelastic analysis in cases where $\sigma(t)$ is prescribed, whereas the generalized Maxwell model is more convenient in cases where the $\varepsilon(t)$ is prescribed. Because of the range of different relaxation times that can be brought into play, both of these models permit a close description of real behavior over a wider time span than is possible with simpler models. Based on Boltzmann's superposition principle, the total strain $\varepsilon(t)$ and stress $\sigma(t)$ can be expressed in the following form:

$$\varepsilon(t) = \int_0^t D(t-\xi)\frac{\mathrm{d}\sigma(\xi)}{\mathrm{d}\xi}\,\mathrm{d}\xi \tag{1}$$

$$\sigma(t) = \int_0^t E(t-\xi)\frac{\mathrm{d}\varepsilon(\xi)}{\mathrm{d}\xi}\,\mathrm{d}\xi \tag{2}$$

Because creep and stress relaxation phenomena are two aspects of the same viscoelastic behavior of materials, they can be related. Applying the Laplace transform to Eqs. (1) and (2) and combining yields the following algebraic equation in transform variable s:

$$\hat{D}(s)\hat{E}(s) = 1/s^2 \tag{3}$$

This is the relation between the creep compliance and relaxation modulus in transform variables for linear materials. If $D(t)$ is known, $E(t)$ can be predicted or vice versa.

In the present study, tensile creep compliance $D(t)$ is evaluated from the tubular tests. The Laplace transform of $D(t)$ is substituted into the preceding equation and solved for $E(s)$. The inverse Laplace transform of $E(s)$ yields the relaxation modulus $E(t)$.

Theoretical and experimental results indicate that for a certain class of material the effect due to time and temperature can be combined into a single parameter through the concept of the time–temperature superposition principle, which implies that the following relation exists:

$$E(T,t) = E(T_0,\zeta) \tag{4}$$

One of the most common functions relating the shift factor and temperature has been proposed by Williams, Landel, and Ferry (WLF) as follows[25]:

$$\log_{10} a_T(t) = \log\frac{t}{\zeta} = \frac{-k_1(T-T_0)}{k_2+(T-T_0)} \tag{5}$$

This WLF equation has been used to describe the temperature effect on relaxation behavior of many polymers with fairly satisfactory results. When this equation was used, the temperature shift factor and material constants of a composite solid propellant were evaluated from different isothermal tubular tests.

Experimental Methods

In this study, the grain pressurization test is called the tubular test. A photograph of the thick-walled tubular specimens (propellant grains) used is shown in Fig. 1a. All of the propellant grains are cast in the same batch at a curing temperature of 60°C for five days. The castings are demolded, tested (nondestructive), accurately cut for a length of 200 mm, and stored horizontally to reduce deformation due to its own weight. The polyvinylchloride casing is removed just before the experiment to reduce environmental effects. The average inner and outer diameters of the grains then measured are 62.2 and 100 mm, respectively.

Uniaxial test specimens are die cut from the propellant blocks, and conventional milling operations are employed for preparing strip biaxial specimens. It is ensured that the samples are not exposed to the atmospheric environment for long periods.

Figures 1b–1d show the shape and dimensions of the tubular, uniaxial, and strip biaxial test specimens. All of the samples are selected from the same batch, and ammonium perchlorate–hydroxyl terminated polybutadiene–aluminium propellant with 18% metal and 86% solid loading is used. The hydroxyl terminated polybutadiene (HTPB) is the fuel in the propellant used.

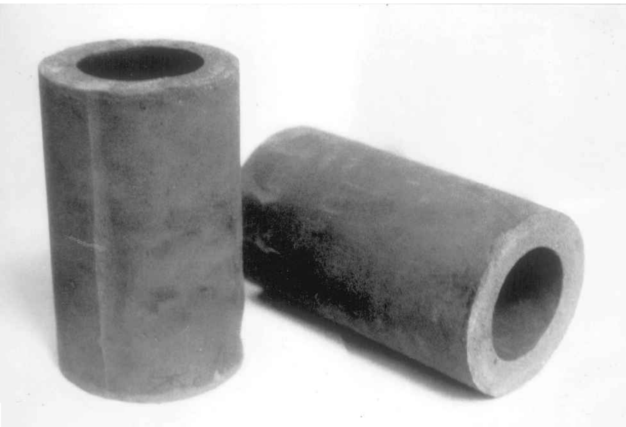

Fig. 1a Photograph of tubular specimens.

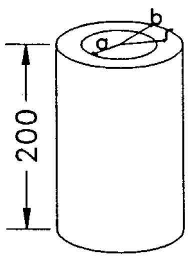

Fig. 1b Tubular specimen.

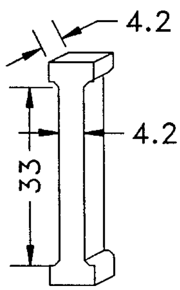

Fig. 1c Uniaxial specimen.

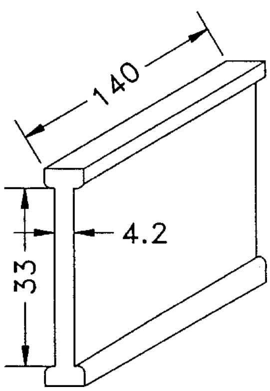

Fig. 1d Strip biaxial specimen.

Stress Relaxation Tests

Both the uniaxial and strip biaxial stress relaxation tests are conducted in an INSTRON 4301 testing machine at 5% strain level at a crosshead speed of 500 mm/minute, and the equation of the relaxation curve fitted is

$$E(t) = \sum_j A(j) \exp\left[-\frac{t}{c}(j)\right] \tag{6}$$

Table 1 Stress relaxation tests data

Uniaxial test		Strip biaxial test	
$A(j)$, kg/cm^2	$C(j)$, s	$A(j)$, kg/cm^2	$C(j)$, s
33.2	19589.6	36.3	31529.8
9.6	221.0	9.7	257.7
11.8	10.3	11.4	13.0

The $A(j)$ and $C(j)$ values of the test specimen are given in Table 1. A computer program is used to enter the test parameters, as well as for data collection and analysis. Tests are carried out at room temperature with appropriate specimen grips.

Tubular Test

The schematic diagram and the photographs of the test facility are presented in Figs. 2a–2c. A $\frac{1}{2}$-in. (1.27 cm) galvanized iron (GI) pipe with two pressure taps and four holes is provided to give compressed air to the inner surface of the specimen. A 6-mm-thick mild steel flange is welded at one end of the GI pipe. One end of the propellant grain is bonded with this flange and the other end to a 10-mm-thick mild steel flange using an adhesive substance (araldite). The wooden frame structure is made of plywood of sufficient thickness to withstand the experimental range of temperature without deformation. Provision is made for the linear expansion of the propellant by permitting free sliding movement of the 10-mm flange over the GI pipe using a metal to metal finish. For easy movement, grease is applied to the sliding surfaces (Fig. 2a).

An electrical motor-driven air compressor (1 hp, 685 rpm, 4–9 atm) with a pressure regulator is used for internal pressurization of the propellant. Two pressure hoses are used to connect the compressor and pressure gauge to the pressure taps of the GI pipe. All joints are made leak proof by using thread seal tape. The entire setup is placed in a temperature-controlled oven with double-paneled glass window as shown in Figs. 2b and 2c. The size of the setup is limited to the available size of the oven (70 × 60 × 60 cm). Electrical heaters are provided in one side of the oven. They are controlled by a thermostat, and the heated air is circulated with the help of a blower. On the other side of the oven, two holes are provided to take out the pressure hoses. Packing is provided in these holes to reduce the heat loss.

During the room temperature test, five dial gauges are used for measuring the outer radial expansion of the propellant, and one dial gauge is used for measuring the axial expansion of the propellant. In the isothermal tests, due to the small size of the viewing window, the dial gauges are limited to three (Figs. 2b and 2c). All of the dial gauges used are able to withstand the experimental range of temperatures with an accuracy of ± 0.002 mm.

Tests are carried out at different temperatures. Propellant specimens are heated with the help of heated air. After reaching the steady-state test temperature, a sudden internal pressure is applied to the tubular grain by opening the compressor outlet valve. Constant pressure is maintained using a regulator. The outer expansion of the tubular specimen is noted at different time intervals until the expansion is almost steady. To obtain the characteristic curves, tests are conducted at different strain levels.

In the analysis, the HTPB-based propellant is considered as a macroscopically homogeneous, thermorheologically simple, and linearly viscoelastic material. Constant specimen volume is also assumed in all of the tests during the interval of interest.

Evaluation of Creep Compliance

From the observed instantaneous variations of the outer radial expansion (Δb) of the propellant and by the use of the incompressibility assumption, the instantaneous inner radial expansion (Δa) of the tubular specimen is computed from the small strain solutions. Also, with use of the basic relation, the corresponding inner-bore hoop strain $\varepsilon_\theta^v(t)$ is calculated. The inner-bore hoop stress σ_θ at different times is computed in accordance with the elastic–viscoelastic correspondence principle.[6] Shear creep compliance $J(t)$ can be

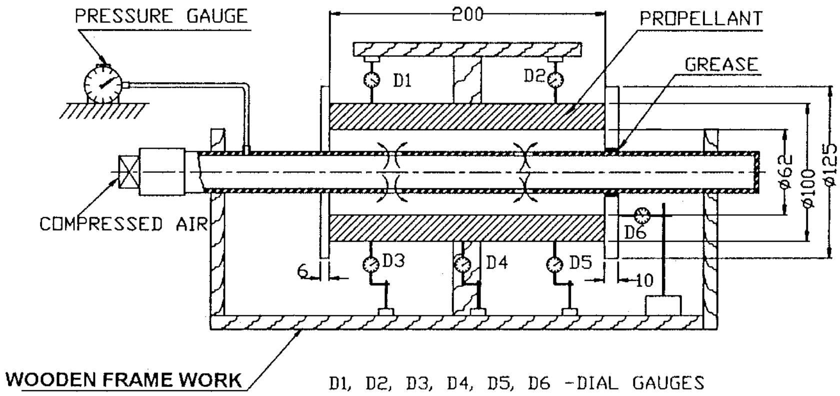

Fig. 2a Schematic of the tubular test facility, all dimensions in millimeters.

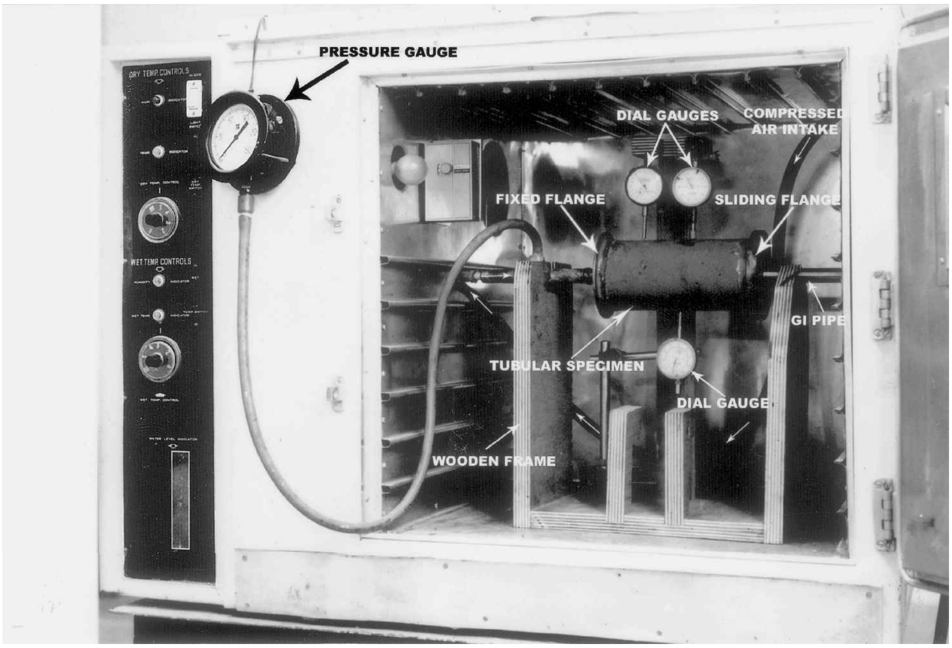

Fig. 2b Photograph of test setup, inside view.

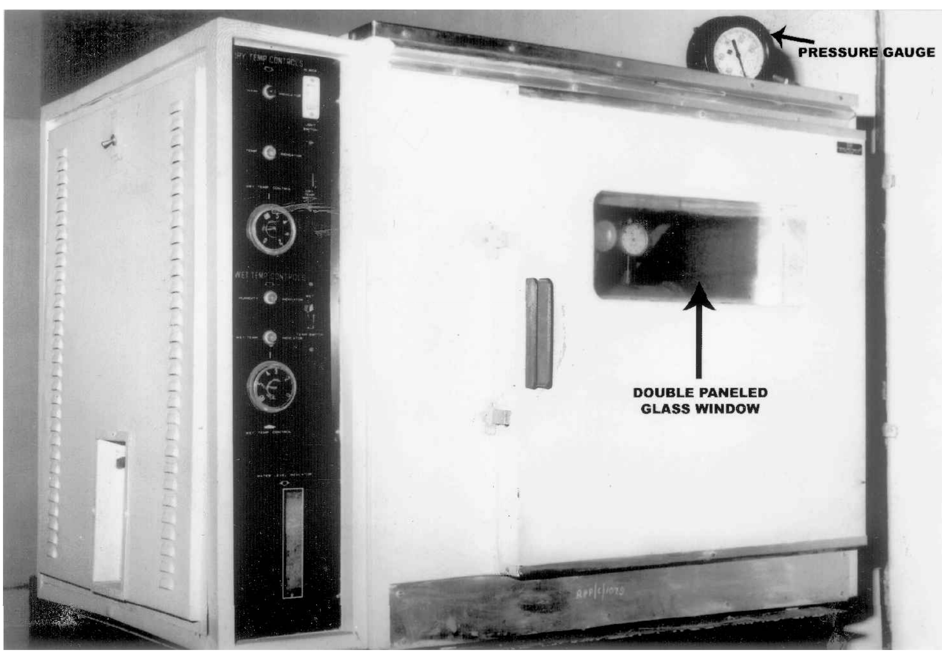

Fig. 2c Photograph of test setup, outside view.

evaluated as

$$J(t) = \frac{\varepsilon_\theta^v(t)}{\sigma_\theta(t)} \tag{7}$$

When generalized plane strain and $\nu = 0.5$ are assumed, the expression for tensile creep compliance may be written as

$$D(t) = J(t)\frac{2(\lambda^2+1)}{3\lambda^2}$$

Evaluation of Relaxation Modulus

All of the tensile creep compliance values obtained from the tubular tests are plotted against time. The best fit curve is found to be a power curve of the form $D(t) = At^n$, where A and n are constants. The constants are found using the least-squares method. The Laplace transform of $D(t)$ becomes

$$\hat{D}(t) = A\Gamma(1+n)/S^{1+n} \tag{8}$$

When Eq. (8) is substituted into Eq. (3), one obtains

$$\hat{E}(s) = [1/A\Gamma(1+n)](1/s^{1-n}) \tag{9}$$

The inverse Laplace transform of Eq. (9) yields the relaxation modulus,

$$E(t) = [1/A\Gamma(1+n)\Gamma(1-n)]t^{-n} \tag{10}$$

This model of course predicts an infinite modulus at $t = 0$. Note that the preceding equation has been used by Lai and Fitzgerald[26] in the prediction of the relaxation modulus of asphalt concrete, and the same model has been successfully applied to a composite solid propellant.[23]

A computer program, AISWARYA, has been developed for computing the $E(t)$ values.

Evaluation of the Time-Dependent Modulus

In accordance with the elastic–viscoelastic correspondence principle, the conventional linear elastic solution with the time-dependent modulus lead to the following equation:

$$E'(t) = \left[1\big/\varepsilon_\theta^e(t)\right][\sigma_\theta(t) - \nu(\sigma_r + \sigma_z)] \tag{11}$$

Where σ_θ, σ_r, σ_Z, and ε_θ^e are evaluated based on elastic solutions. The mechanical testing laboratory reported that a given specimen from the same batch demonstrated linearity up to a 10% strain level[23] that and up to this strain level, $\nu = 0.5$.

An effort has also been made to compute the viscoelastic hoop strain from the predicted relaxation-modulus values of a tubular specimen using the following relation for comparison:

$$\varepsilon_\theta^v = [1/E(t)][\sigma_\theta(t) - \nu(\sigma_r + \sigma_z)] \tag{12}$$

A computer program, CJR, has been developed for computing $E'(t)$ and ε_θ^v values.

Evaluation of Temperature Shift Factor

The horizontal shift from the relaxation curve at the reference temperature to the other two curves gives the values of the corresponding shift factors. When the two simultaneous equations obtained by substituting the values of a_T in Eq. (5) are solved, the material constants k_1 and k_2 are evaluated. Good temperature control is necessary for this study, because temperature fluctuation will cause fluctuation in thermal expansion, and thermal expansion is difficult to isolate from creep strains.

Results and Discussion

In the isothermal tubular tests, due to the small size of the viewing window in the oven, parallax errors crept in during the observations of the dial gauge readings. Errors are minimized when the dial gauges are kept inline with the viewing window. Radial expansion obtained from the diametrically opposite dial gauges, kept at the middle of the specimen, are used for the computation. Hence, errors due to end effects are safely omitted. Misalignments of the dial gauges were also noticed in some of the isothermal tests because of the shock produced during the closing of the oven door. Variations of the initial settings of the dial gauges after closing the door have been appropriately adjusted with the final readings of the dial gauges. Observed temperature variations of $\pm 0.5^\circ$C is neglected.

From the mechanical testing laboratory data, the propellant sample shows linearity up to 10% strain level. In the reported tubular tests, the maximum strain level reached is 11.04%. Hence, the linear viscoelastic characterization for the test condition is justifiable. Because of the small size of the viewing window of the oven, separate tests were carried out to observe the linear expansion of the tubular specimen at different temperatures. It was observed that the linear expansion is negligibly small compared to the radial expansion.

Test data revealed that the time to reach the nonlinearly is on the order of seconds. The slight nonlinearly is observed only after 1410 s in the fourth test. From the ballistician's point of view, the response time of the HTPB-based propellant to reach the nonlinearity is very long. Note that in the actual case the ignition peak will take place within fraction of a second in a solid rocket motor and that the total burning time will be in the order of seconds. Hence; it can be safely concluded that the nonlinearity will not be attained during the starting transient of solid rocket motors with that propellant batch. However the possibility to attain the nonlinearity during the web burning time may be examined separately. In the case of a long duration, web burning motors particularly for missiles memory effect due to pressure overshoot (ignition peak) coupled with elevated pressure and temperature may lead to nonlinearity. This is an area that is to be examined more carefully.

The variation of hoop stress with hoop strain at different test conditions is shown in Fig. 3. At small strain levels (1.6–2.4%), the 15.6% increase in the oven temperature causes the strain level range to increase from 2.1 to 2.82%. An increase of 25% in both the internal pressure and temperature increases the strain level range from 3.8–5.91% to 9.03–11.04%.

The shear creep compliance data obtained from the tubular tests are transformed into tensile creep compliance and plotted in Fig. 4. Significant variation of creep compliance is noticed at the higher temperature test. This indicates that, when the specimen temperature is relatively higher than the room temperature, the tendency to show nonlinearity will be at a shorter interval of time. In the fourth test, nonlinearity is evident after 3200 s.

In all of the cases, the best fit creep compliance curve is obtained in the form of a power curve, $D(t) = A\,t^n$. This simple form of

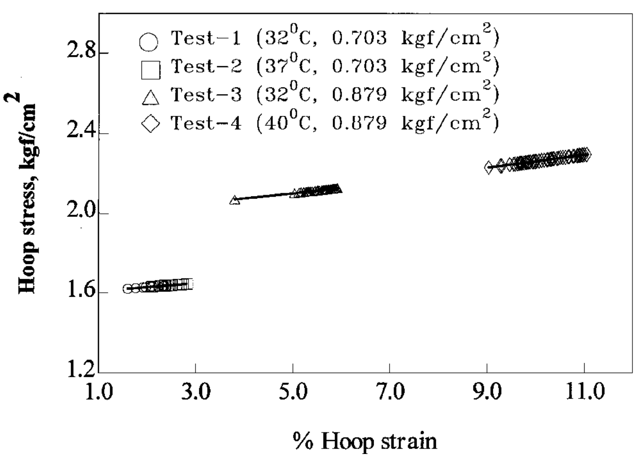

Fig. 3 Variation of hoop stress with hoop strain at different tubular test conditions.

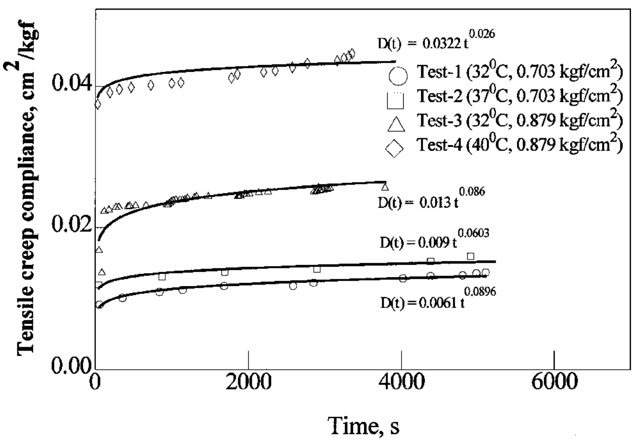

Fig. 4 Comparison of the tensile creep compliance curves obtained at different test conditions.

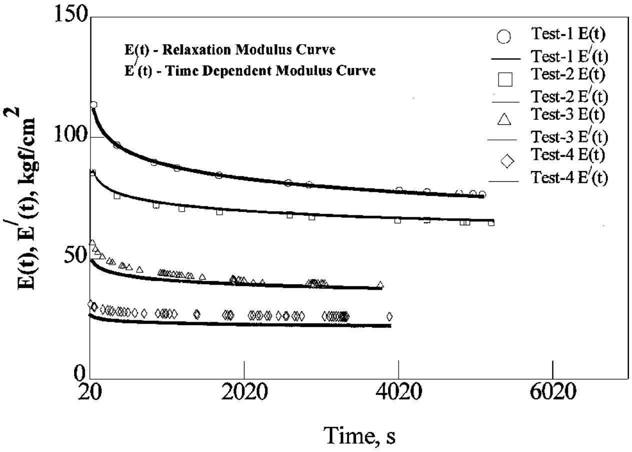

Fig. 6 Comparison of the experimentally predicted relaxation modulus and theoretically predicted time-dependent modulus.

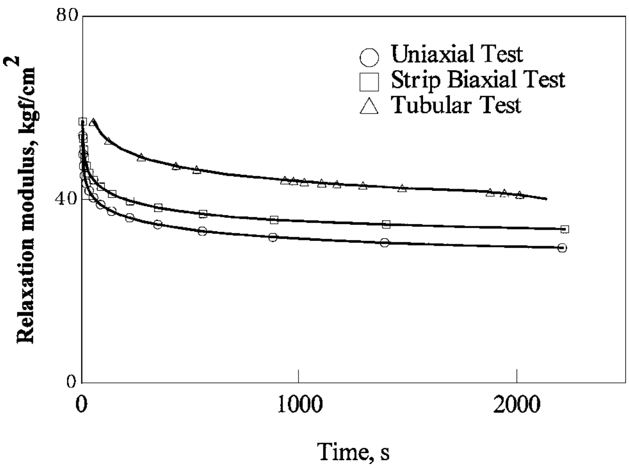

Fig. 5 Comparison of the relaxation-modulus curves obtained from three different methods.

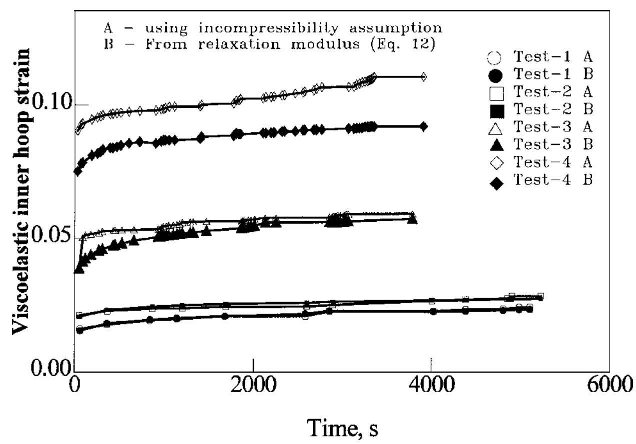

Fig. 7 Comparison of the viscoelastic inner hoop strain computed using the incompressibility assumption and using Eq. (12).

tensile creep compliance made it possible to predict the $E(t)$ values of the propellant using Eq. (10). The predicted $E(t)$ values of the tubular test ($P_i = 0.879$ kgf/cm^2) is compared with the $E(t)$ values obtained from the uniaxial and strip biaxial tests at 5% strain level. Figure 5 shows the comparison of the relaxation-modulus values obtained with the three different methods.

It has seen that, during the initial period, the relaxation-modulus curves are close together. Later the relaxation modulus obtained from tubular test is 25–43% higher than the uniaxial test and 18–30% higher than the strip biaxial test. This is because in the initial stage the stress–strain ratios were almost same and the stress conditions vary subsequently according to the geometry of the specimen. It may also be concluded that the higher values of relaxation modulus obtained in the tubular test are due to the multiaxial stress condition in the specimen. The data obtained from the tubular test may be considered more realistic because the test conditions are closer to the actual condition.

Effort has also been made to predict the time-dependent modulus using the correspondence principle. Comparison of the experimentally predicted relaxation-modulus curves and theoretically predicted time-dependent modulus curves are presented in Fig. 6. Note that in the first two tests the relaxation-modulus curves and the time-dependent modulus curves are closer together, but in the remaining two tests, $E(t)$ values are found to be 1–7% higher than the $E'(t)$ values.

From this analysis one can conclude that, for many practical long time linear problems, accuracy of about 7% is more than enough, and it is reasonable to treat the tensile creep modulus as a time-dependent elastic modulus. In such cases, the elastic–viscoelastic correspondence principle can be effectively applied for viscoelastic characterization. In this study, the solution technique has been focused on using the elastic–viscoelastic correspondence principle. This method is by far the simplest method, although as mentioned in the Introduction, not all linear viscoelastic problems can be solved by this method. In all of the tests, the relaxation-modulus and time-dependent modulus are decreasing with increasing time, and significant variation of relaxation-modulus values are observed at the higher internal pressure and specimen temperature.

In another verification attempt, the viscoelastic inner hoop strain has been computed using Eq. (12) and compared with the corresponding values obtained through the incompressibility assumption. In all of the cases, it was observed that there is a sudden change in the inner viscoelastic hoop strain for a very short interval and a gradual increase in the remainder of the test periods. At a higher strain level (tests 3 and 4), significant differences in hoop strain are also observed. This can be seen in Fig. 7.

To provide a reference of the test results to temperature, the WLF superposition principle is used. The relaxation modulus curves shown in Fig. 6 were extrapolated, and a shift factor was evaluated with respect to a reference temperature. Material constants are derived after solving the WLF equation. Figure 8 shows the variation of strain rates with time at different experimental conditions.

Stress–strain values obtained from different tubular tests are superposed into a master stress–strain curve, which is shown in Fig. 9. This curve describes the response of the material over a very wide range of temperature-reduced strain rates. Figure 10 shows the master stress relaxation-modulus curve, and Fig. 11 shows the master relaxation-modulus–tensile creep compliance curve. In all of the cases, $\varepsilon/\dot{\varepsilon}$ is the time required to achieve a strain ε at a constant strain rate $\dot{\varepsilon}$. This time value after translation to the desired temperature has

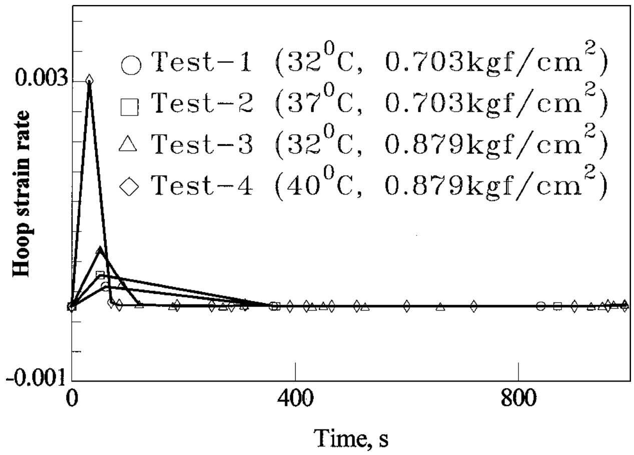

Fig. 8 Variation of viscoelastic hoop strain rate with time at different experimental conditions.

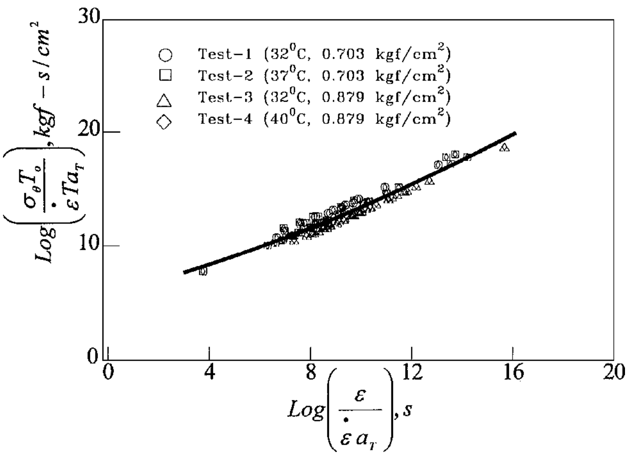

Fig. 9 Master stress–strain curve obtained from tubular tests.

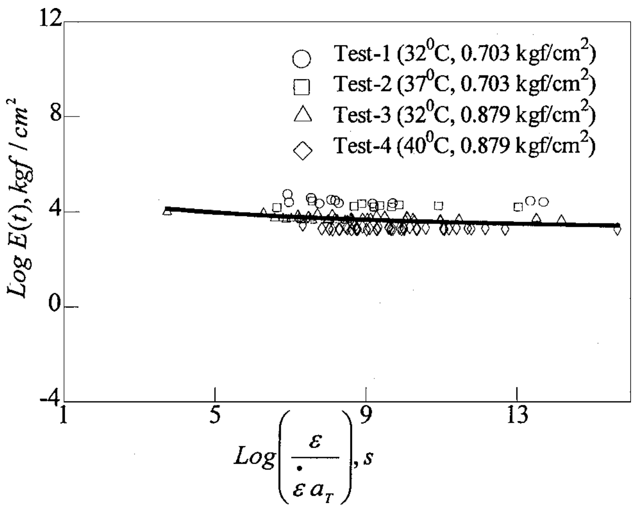

Fig. 10 Master stress relaxation modulus curve obtained from tubular tests.

been used in plotting the master relaxation-modulus curve. When the strain rate and the shift factor are used, different characteristics curves can be generated.

Note that the main objective of this characterization was not to determine the magnitude of the stress actually present in the propellant, but to help the designer to decide the best configuration from the stress relaxation point of view.

The results reported are essentially to prove the merits of the tubular tests over other available techniques. To obtain more accurate data, instead of using the incompressibility assumption, it is suggested to measure the inner hoop strain directly using strain gauges. Having proved the merits of the tubular test, the next step will be to standardize this method for characterizing solid propellants in general.

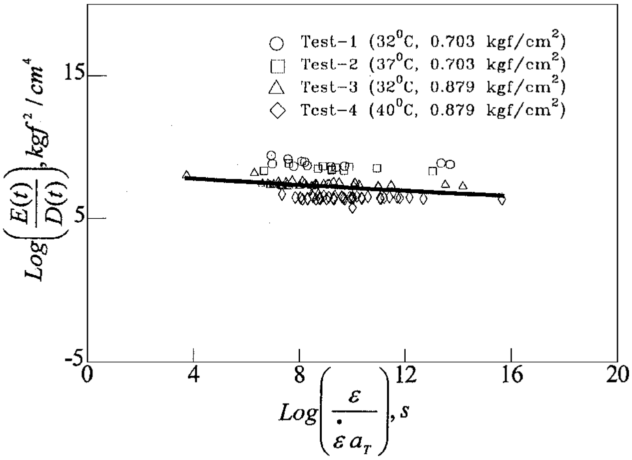

Fig. 11 Master relaxation modulus–tensile creep compliance curve obtained from tubular tests.

Conclusions

In the design of solid-propellant grains, the relaxation modulus plays a key role. It is the stress per unit of applied strain and is different for each material. Propellant with a higher relaxation modulus will develop a crack on the surface of the grain during the actual firing condition. For those propellants with low relaxation modulus, the grain will deform during demolding and storing periods. It will also cause a change in shape of the inner surface of the grain during the launching periods. The configuration of the inner surface of the grain is determined mainly by ballistic considerations. Hence, more realistic prediction of relaxation-modulus values of solid propellants are essential for the design of a solid rocket motor from the point of view of stress relaxation. The tubular test appears to be a more acceptable test than uniaxial and strip biaxial tests for characterizing solid propellants because it is more realistic. The tubular test with internal pressurization is a more convenient and realistic method for predicting relaxation-modulus values of solid propellants and has great potential for characterizing propellants in general.

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